Distributed Electromagnetic Component Sensor Array Processing

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1 Abstract

We propose an approach to localize multiple sources based on spatially distributed electric and magnetic component sensors. This approach exploits all available directional information in the electromagnetic field and differential time delay for direction of arrival (DOA) estimation. As a result, it can outperform both a single vector-sensor and scalar-sensor arrays in accuracy of direction of arrival (DOA) estimation.
2 MOTIVATION

- Most existing array processing methods rely on the spatial diversity of the sensor array to estimate the source direction of arrival (DOA).

- A drawback of this approach is that the accuracy performance becomes highly dependent on the electrical aperture of the array.

- To avoid ambiguities in the array manifold, the physical size of such broadband arrays is constrained by the highest operating frequency.

- Poorer performance at lower frequencies will result due to their larger wavelengths especially when the number of receiver channels is limited.

- Increasing the number of receiver channels to achieve larger “unambiguous” array geometry and size is a costly approach.

Hence, there is a need to develop DOA estimation methods that use small aperture array that achieve good performance over a wide operating frequency.
Component sensors distributed as an array of scalar magnetic and electric sensors. It is assumed that the array of scalar magnetic and electric sensors should, in aggregate, measure the all the 3D electric and magnetic components of the electromagnetic wave.

Advantages:

- Full electric and magnetic field components measure by the magnetic and electric sensors can be used to derive the sources’ directional information.

- Additional sources’ directional information can be obtained from the differential-delay measurements.

- Significantly smaller number of receivers to simultaneously utilize the time-delay and complete electromagnetic information to perform DOA estimation.
4 MEASUREMENT MODEL

Let

\[
(u \times) = \begin{bmatrix}
0 & -u_z & u_y \\
u_z & 0 & -u_z \\
-u_y & u_x & 0
\end{bmatrix}
\]  \hspace{1cm} (1)

where

- \( u \): unit direction vector from sensor to source
- \( u_x, u_y \) and \( u_z \): \( x, y \) and \( z \) components of \( u \).

Also let

\[
V = \begin{bmatrix}
-\sin \theta_1 & -\cos \theta_1 \sin \theta_2 \\
\cos \theta_1 & -\sin \theta_1 \sin \theta_2 \\
0 & \cos \theta_2
\end{bmatrix}, \hspace{1cm} (2)
\]

\[
Q = \begin{bmatrix}
\cos \theta_3 & \sin \theta_3 \\
-\sin \theta_3 & \cos \theta_3
\end{bmatrix}
\]  \hspace{1cm} (3)

and

\[
w = \begin{bmatrix}
\cos \theta_4 \\
j \sin \theta_4
\end{bmatrix}
\]  \hspace{1cm} (4)

where \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4 \) are the azimuth, elevation, ellipse’s orientation and ellipticity angle.
Assuming narrowband EM sources, the measurement model of the distributed component sensor array in multiple sources environment:

\[
\begin{bmatrix}
    y_E(t) \\
    y_H(t) \\
    y(t)
\end{bmatrix} = \sum_{k=1}^{d} a(\theta^{(k)}) s_k(t) + \begin{bmatrix}
    e_E(t) \\
    e_H(t) \\
    n(t)
\end{bmatrix}, (5)
\]

\[a(\theta^{(k)}) = \Gamma(\theta_1^{(k)}, \theta_2^{(k)})\Omega \begin{bmatrix}
    I_3 \\
    (u_k \times)
\end{bmatrix} V_k Q_k w_k\]

where

- \(\theta^{(k)} = [\theta_1^{(k)}, \theta_2^{(k)}, \theta_3^{(k)}, \theta_4^{(k)}]\).
- \(\Gamma(\theta_1, \theta_2)\) : diagonal matrix where \(n\)-th diagonal entry is given by \([\Gamma(\theta_1, \theta_2)]_{nn} = a_n(\theta_1, \theta_2)e^{j\omega_c \tau_n}\).
- \(\tau_n\) : differential delay of signal source between the \(n\)-th component and the phase center.
- \(a_n(\theta_1, \theta_2)\) : response of the \(n\)-th component sensor. \(\omega_c\) : carrier frequency.
- \(\Omega\) : selection matrix of 1 and 0.
Observe that the electromagnetic sources directional information are all embedded in
\[ \Gamma(\theta_1^k, \theta_2^k) \Omega \begin{bmatrix} I_3 \\ (u_k \times) \end{bmatrix} V_k. \]

**Remark 1** This allows joint exploitation of differential delay measurements & EM field measurements for source parameter estimation.

Express (5) compactly in matrix form:
\[ y(t) = As(t) + n(t) \] (6)
where
\[ A = \begin{bmatrix} a(\theta^{(1)}) \cdots a(\theta^{(d)}) \end{bmatrix} \] (7)
and \[ s(t) = [s_1(t) \ldots s_d(t)]^T. \]
5 Numerical Example

Simulation Parameters:

- Assume a six-channel receiver and use a 6 element uniform circular array.
- Inter-element spacing is fixed at $\frac{c}{2f_{\text{max}}}$ where $c$ is the speed of light and $f_{\text{max}}$ is the maximum operating frequency.
- Two uncorrelated signal sources. Signal to Noise Ratio: 10dB.
- Array Configuration under consideration:
1. Array Geometry of Distributed EM Component Sensor Array, x-electric (co-polarized) array and electric-only diversely polarized array. $E_x(H_x)$, $E_y(H_y)$ and $E_z(H_z)$ are the electric (magnetic) component sensors.
Example 1  DOA estimation performance as a function of frequency. Two uncorrelated sources with $\mathbf{\Theta}^{(1)} = [1^\circ, 10^\circ, 45^\circ, 0^\circ]^T$ and $\mathbf{\Theta}^{(2)} = [5^\circ, 9^\circ, -45^\circ, -5^\circ]^T$. 
Example 2  DOA estimation performance as a function of azimuth angle of separation between uncorrelated two sources of 10 dB SNR. Normalized operating frequency is fixed at $\frac{f}{f_{\text{max}}} = 0.3$. 
6 CONCLUDING REMARKS

- Presented a new approach for the localization of electromagnetic sources by joint exploitation of the spatial diversity and electromagnetic information using spatially distributed electric and magnetic component sensors.

- Performance analysis via numerical examples illustrated the potential gain of the proposed approach over scalar and diversely polarized array.

- Analysis indicated that the proposed distributed component EM sensor array should allow the use of small array apertures while maintaining desired resolution and accuracy performance over a wide operating bandwidth.
3. CRB vs Angular Separation