Adaptive Formation of Sum and Difference Beams to Maintain Monopulse Slope

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Why Adaptive Processing?

Problem: Bistatic Space-Based Radar Must Detect and Locate Moving Ground Targets

- STAP required for slowly-moving targets
- Monopulse slope after adaptation is distorted unless constraints are applied
Definitions

N = Number of antennas
M = Number of time taps/antenna
T = Pulse repetition interval
x_n = Location of antenna n in linear array
w = Sum-beam weight vector (NM x 1)
w_Δ = Difference-beam weight vector (NM x 1)
g(θ, f) = Vector (NM x 1) response to point target at angle θ with Doppler f. Typical component is

\[ g_{nm} = \exp \left( \frac{i2\pi x_n}{\lambda} \sin \theta - i2\pi fmT \right) \]

\[ V_\Sigma(\theta, f) = w^T g(\theta, f) = \text{sum - beam voltage} \]
\[ V_\Delta(\theta, f) = w_\Delta^T g(\theta, f) = \text{difference - beam voltage} \]
Procedure

1) Form adapted sum beam in conventional manner using weight vector

\[ w = \mu R^{-1} s^* \]  \hspace{1cm} (1)

\( R = \) Interference covariance matrix
\( s = \) Steering vector (function of target angle and Doppler)
\( \mu = \) Normalization constant

2) Constrain monopulse slope at K points. For example, if \( K = 2 \)

\[ \frac{V_{\Delta}(\theta_o \pm \Delta \theta, f)}{V_{\Sigma}(\theta_o \pm \Delta \theta, f)} = \pm k_s \Delta \theta \]  \hspace{1cm} (2)

\( k_s = \) slope constant
Procedure (Concluded)

3) Substitute for $V_{\Delta}$ and $V_{\Sigma}$ to rewrite (2) as

$$g^T (\theta_o + \Delta \theta, f) w_{\Delta} = k_s \Delta \theta \quad w^T g(\theta_o + \Delta \theta, f)$$
$$g^T (\theta_o - \Delta \theta, f) - w^T g(\theta_o - \Delta \theta, f)$$

4) Generalize Equation (3) to $K$ constraints as

$$H^T w_{\Delta} = \rho$$

5) Minimize the difference-beam interference

$$w_{\Delta}^H R w_{\Delta}$$

subject to constraint in (4). Solution is

$$w_{\Delta} = R^{-1} H^* \left( H^T R^{-1} H^* \right)^{-1} \rho$$

Note: different weight vector for each target Doppler
Results

- Procedure applied to linear array of 13 adaptive subarrays, each processing 14 pulses (182 degrees of freedom)

- Clutter cancelled from 62 dB above noise down to very near noise floor in both sum and difference beams for all targets with speeds from 0.02 * blind speed to 0.98 * blind speed

- Monopulse slope maintained nearly linear with angle for all target speeds in the above range

- Typically only 3 constraints necessary (at $\theta = \theta_0$, $\theta_0 - \Delta \theta$, $\theta_0 + \Delta \theta$)
Positive Angle Portion of Adapted Monopulse Pattern for Three Constraints