QUADRATICALLY CONSTRAINED RLS FILTERING
for ADAPTIVE BEAMFORMING
and DS-CDMA MULTI-USER DETECTION

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Beamforming and DS-CDMA

- Exploit equivalence between
  - Minimum Output Energy (MOE) detector for Direct-Sequence Code Division Multiple Access (DS-CDMA) wireless communications systems
  - Linearly Constrained Minimum Power (LCMP) beamformer

- In both applications, a quadratic constraint on the weight vector norm can improve robustness to mismatch and low sample support
Variable Loading RLS

- Developed technique for implementing quadratic inequality constraint with Recursive Least Squares (RLS) updating

- RLS-VL has fast convergence and better performance than other RLS and Least Mean Square (LMS) implementations for both beamforming and DS-CDMA
DS-CDMA Signal Model

\[ c_1 = [-1 \ 1 \ -1 \ 1]^T \]

User 1

\[ T_s \]

\[ T_c \]

\[ c_2 = [1 \ 1 \ -1 \ -1]^T \]

User 2

bits  code  chips
DS-CDMA Signal Model

$M$-user, synchronous, DS-CDMA binary communications system in additive white Gaussian noise (AWGN) channel.

- Received baseband signal model:

$$x(t) = \sum_{k=-\infty}^{\infty} \sum_{m=1}^{M} A_m b_m(k) \left\{ \sum_{l=0}^{L-1} c_m(l) \psi(t - kT_s - lT_c) \right\} + \sigma_n n(t)$$

- $A_m$: amplitude of $m$th user’s signal
- $b_m(k)$: $k$th data bit of $m$th user
- $c_m(l)$: $l$th chip $m$th user’s code
- $L$: number of chips per symbol (processing gain)
- $T_c$: chip interval
- $T_s$: symbol interval ($= LT_c$)
- $\psi(t)$: chip waveform with support on $[0, T_c]$
- $n(t)$: AWGN with unit power spectral density
- $\sigma_n^2$: noise power spectral density
DS-CDMA Signal Model

- DS-CDMA data vector after matched filtering and sampling at chip rate

\[
x(k) = \sum_{m=1}^{M} A_m b_m(k)c_m + \sigma_n n(k)
\]

- Array processing data vector

\[
x(k) = \sum_{m=1}^{M} s_m(k)v_m + \sigma_n n(k)
\]

- Code vector \(c_m\) equivalent to array response vector \(v_m\)
- Source waveform is scaled \(k\)th data bit \(A_m b_m(k)\)
Challenges in DS-CDMA Wireless Communication Systems

- Co-channel multiple-user interference (CCI/MUI)
- Multi-path time-varying propagation environment
- Increase capacity by increasing throughput while allowing more users at lower and variable signal levels
Minimum Output Energy (MOE) Detector
(Honig, Madhow, Verdu (95), Schodorf and Williams (97))

- Optimization Problem
  \[
  \min \ w^H R_x w \quad \text{st. } C^H w = f
  \]

- Solution
  \[
  w = R_x^{-1} C \left[ C^H R_x^{-1} C \right]^{-1} f.
  \]

- MOE: \( C = c_1, f = 1 \)

- Decorrelating Detector: \( C = [c_1 \ c_2 \ \cdots \ c_K], \ f = [1 \ 0 \ \cdots \ 0]^T \)

- Blind Implementation
Partitioned Linear Interference Canceler (PLIC)
(Schodorf and Williams (97))

- Equivalent to Generalized Sidelobe Canceler (GSC)
  (Griffiths and Jim (82))
Partitioned Linear Interference Canceler (PLIC)

- **Total weight vector** $L \times 1$
  \[ w = w_c - Bw_a. \]

- **Quiescent weight vector** $L \times 1$
  \[ w_c = C(C^H C)^{-1}f \]

- **Blocking matrix** $L \times (L - q)$
  \[ B^H C = 0, \quad B^H B = I \]

- **Adaptive weight vector** $(L - q) \times 1$
  \[ w_a = (B^H R_x B)^{-1}B^H R_x w_c \]
  \[ = R_z^{-1} p_z \]
Quadratic Constraints and Robustness

- A quadratic inequality constraint on the weight vector norm of an adaptive MOE detector can improve robustness to mismatch in temporal signature vector due to multipath propagation

- In adaptive beamforming, the quadratic inequality constraint (white noise gain constraint) improves robustness to mismatch in steering vector due to pointing error and sensor perturbations
Quadratically Constrained MOE

- Quadratic constraints
  \[ w^H w = w_c^H w_c + w_a^H B^H B w_a \leq T_0 \]
  \[ w_a^H w_a \leq \beta^2 = T_0 - w_c^H w_c \]

- Optimization problem
  \[ \min w^H R_x w \quad \text{st.} \quad C^H w = f, \quad w^H w \leq T_0 \]

- Direct form solution
  \[ w = (R_x + \lambda I)^{-1} C \left[ C^H (R_x + \lambda I)^{-1} C \right]^{-1} f \]

- PLIC/GSC solution
  \[ w_a = (R_z + \lambda I)^{-1} p_z \]

- Same forms with a diagonal loading term added to \( R_x \) and \( R_z \)
Diagonal Loading

- Let \( \tilde{w}_a \) denote standard PLIC/GSC adaptive weight vector
  \[
  \tilde{w}_a = R_z^{-1} = \sum_{i=1}^{L-q} u_i \frac{u_i^H p_z}{\lambda_i}
  \]

- Let \( w_a \) denote quadratically constrained weight vector
  \[
  w_a = \sum_{i=1}^{L-q} u_i \frac{u_i^H p_z}{\lambda_i + \lambda} = \sum_{i=1}^{L-q} u_i \frac{\lambda_i}{\lambda_i + \lambda} \frac{u_i^H p_z}{\lambda_i}
  \]

  Each orthogonal component is scaled back by \( \lambda_i / (\lambda_i + \lambda) \)

- Optimal Diagonal Loading
  - For \( \lambda = 0 \), \( w_a = \tilde{w}_a \) (standard MOE adaptive weights)
  - For \( \lambda \rightarrow \infty \), \( w_a = 0 \) (quiescent weights)
  - Can show that \( w_a^H w_a \) is monotonically decreasing in \( \lambda \)
  - Start with \( \lambda = 0 \), increase until quadratic constraint met
Implementing Quadratic Constraint in MOE Detector

- No closed form solution for optimal loading level
- Can fix $\lambda$ at reasonable level, but constraint will not always be met
- Optimal loading level depends on scenario, while a constant norm constraint can work well over a range of scenarios
- LMS implementations
  - LMS with Fixed Loading (Honig, Madhow, Verdu (95))
  - LMS with Scaled Projection (Schodorf and Williams (97) based on Cox, Zeskind, Owen (87))
- No RLS implementations with quadratic constraint
  - RLS without quadratic constraint (Poor and Wang (97))
LMS Implementations

- **LMS Scaled Projection**

  \[ \tilde{w}_a(k) = w_a(k - 1) + \alpha z(k) (y_c^*(k) - z(k)^H w_a^H(k - 1)) \]

  if \[ ||\tilde{w}_a(k)||^2 \leq \beta^2 \], then \[ w_a(k) = \tilde{w}_a(k) \]

  if \[ ||\tilde{w}_a(k)||^2 > \beta^2 \], then \[ w_a(k) = \tilde{w}_a(k) \frac{\beta}{||\tilde{w}_a(k)||} \]

- **LMS Fixed Loading**

  \[ w_a(k) = (I - \alpha \lambda) w_a(k - 1) + \alpha z(k) (y_c^*(k) - z(k)^H w_a^H(k - 1)) \]
**RLS Variable Loading**

*(Tian, Bell, Van Trees (98))*

- Rewrite adaptive weight vector
  
  \[ w_a = (I + \lambda R_z^{-1})^{-1} R_z^{-1} p_z = (I + \lambda R_z^{-1})^{-1} \tilde{w}_a \]

- For small \( \lambda \)
  
  \[ w_a \approx (I - \lambda R_z^{-1}) \tilde{w}_a = \tilde{w}_a - \lambda v_a \]

  where

  \[ v_a = R_z^{-1} \tilde{w}_a \]

- Each component scaled back by \( 1 - (\lambda/\lambda_i) \)

- Quadratic constraint yields closed form solution for \( \lambda \)

\[ w_a^H w_a = \tilde{w}_a^H \tilde{w}_a - 2 \lambda \Re \{ \tilde{w}_a^H v_a \} + \lambda^2 v_a^H v_a \leq \beta^2 \]
RLS Implementation

- RLS algorithm updates unconstrained $\tilde{w}_a(k)$ and an estimate of $R_z^{-1}$, denoted by $P(k)$

\[
g(k) = \frac{\mu^{-1}P(k-1)z(k)}{1 + \mu^{-1}z^H(k)P(k-1)z(k)}
\]

\[
P(k) = \mu^{-1}P(k-1) - \mu^{-1}g(k)z^H(k)P(k-1)
\]

\[
e_p(k) = y_c(k) - w_a^H(k-1)z(k)
\]

\[
\tilde{w}_a(k) = w_a(k-1) + g(k)e^*_p(k)
\]
RLS Variable Loading Implementation

if \( \|\tilde{\mathbf{w}}_a(k)\|^2 \leq \beta^2 \)

\[
\mathbf{w}_a(k) = \tilde{\mathbf{w}}_a(k)
\]

else

\[
\mathbf{v}_a(k) = \mathbf{P}(k)\tilde{\mathbf{w}}_a(k)
\]

\[
a = \|\mathbf{v}_a(k)\|^2
\]

\[
b = -2\Re \{\mathbf{v}_a(k)^H \tilde{\mathbf{w}}_a(k)\}
\]

\[
c = \|\tilde{\mathbf{w}}_a(k)\|^2 - \alpha^2
\]

\[
\lambda(k) = \frac{-b - \Re \{\sqrt{b^2 - 4ac}\}}{2a}
\]

\[
\mathbf{w}_a(k) = \tilde{\mathbf{w}}_a(k) - \lambda(k)\mathbf{v}_a(k)
\]

end.
Beamforming Example: Pointing Error
(from Tian, Bell, Van Trees (98))

- 10 element ULA, $d = \lambda/2$, steered to the broadside
- Source at $\cos(\theta_s) = -0.03$, SNR = 10dB
- Two interferers at $\cos(\theta_1) = 0.29$ and $\cos(\theta_2) = 0.45$, INR = 20dB
- Tolerance factor $T_o = 2/N$
DS-CDMA Example: Multipath

- 15 dB Desired user with $L = 31$ Gold Code
- Seven 15 dB interferers, new 25 dB interferer added at $k = 500$
- Dominant path plus one multipath delayed by 3 chips
- Tolerance factor $T_o = 2/L$
DS-CDMA Example: Multipath

- Desired user with $L = 31$ Gold Code
- Seven interferers with INR=SNR
- One interferer with INR = SNR+10 dB
- Dominant path plus one multipath delayed by 3 chips
- Tolerance factor $T_o = 2/L$
Summary

- Developed RLS-VL technique to implement quadratic constraint with RLS
  - Provides variable, approximate diagonal loading
  - Does not increase contributions significantly

- Applied RLS-VL to MOE DS-CDMA detector by exploiting equivalence with LCMP beamformer
  - Improves robustness in multipath environment
  - Converges faster than robust LMS versions