Equalization of OFDM under Time-Varying Channel Conditions

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Outline

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    ▪ Transmitter
    ▪ Receiver – delay-only
  – Problem Definition
    ▪ Delay-Doppler Channel Model
    ▪ Prior Work

• Extension of OFDM for large Doppler spread
  – MMSE-based Receiver
  – Simulated Performance for complete channel information

• Parameter Estimation
  – Channel Model
    – Uncoupled estimation of Doppler, Delay, and Gain

• Conclusions & Future Work
Orthogonal Frequency Division Modulation (OFDM) – a waveform, TDMA in nature, designed to accommodate multi-path time dispersion

- Cyclic extensions avoid ISI due to delay spread & aid or enable synchronization
- Cyclic Prefix generated by copying end of \( x(n) \) to beginning
  - Suffix – add beginning to end
- Windowing reduces spectral growth
**The OFDM Transceiver**

- A transmit time-domain sequence \( x(n) \) is produced from transmit symbol vector \( \bar{s} \):

\[
\bar{x} = C_p W^* \bar{s}
\]

where \( W^* \) is the IFFT matrix, \( C_p \) the cyclic prefix operator, \( N \) the FFT length and \( L \) the number of samples in \( C_p \):

\[
C_p = \begin{bmatrix}
I_L \\
I_N
\end{bmatrix}
\]
The OFDM Receiver -- handles channel time dispersion in a computationally efficient manner

- Representing the channel \( h(n) = \sum_{i=1}^{K} \alpha_i \delta(n - \tau_i) \), having \( K \) paths, in delay-operator matrix form

\[
D_{\tau_i} = W^* = \begin{bmatrix}
    e^{-j\tau_i} & 0 & \cdots & 0 \\
    0 & e^{-j\tau_{i,1}} & 0 & \cdots \\
    \vdots & 0 & \ddots & 0 \\
    0 & \cdots & 0 & e^{-j\tau_i(N-1)}
\end{bmatrix} W
\]

\[
D_\tau = \left( \sum_{i=1}^{K} \alpha_i D_{\tau_i} \right)
\]

- The received signal \( r(n) \) is then given by:

\[
\tilde{r} = C_{pr} D_\tau C_p W^* \tilde{s} \quad \quad \quad C_{pr} = \begin{bmatrix} 0 & I_N \end{bmatrix}
\]

where \( C_{pr} \) removes cyclic prefix

- Channel does not destroy orthogonality of signaling basis functions

\[
\tilde{s} = W \tilde{r} \quad \quad \quad \hat{s} = \hat{H}^* W \tilde{r}
\]

where \( \hat{H} \) is a diagonal matrix containing the channel frequency response, enabling coherent detection
Consider the general delay-Doppler channel model

- The signal $y(n)$ at channel output given transmit signal $x(n)$ at channel input:

$$y(n) = \sum_{i=1}^{K} \alpha_i e^{j\Omega_i (n-\tau_i)} x(n-\tau_i) + \eta(n)$$

having independent Doppler $\Omega_i$ and Delays $\tau_i$ on $K$ separate “rays” and assumed to be constant over the OFDM Frame

- The channel is given in *delay-Doppler* operator form by summing over each of the $K$ paths:

$$D_{\Omega} = \left( \sum_{i=1}^{K} \alpha_i D_{\tau_i} D_{\Omega_i} \right)$$

$$D_{\Omega_i} = \begin{bmatrix}
e^{j\Omega_{i,0}} & 0 & \cdots & 0 \\
0 & e^{j\Omega_{i,1}} & 0 & \vdots \\
\vdots & 0 & \ddots & 0 \\
0 & \cdots & 0 & e^{j\Omega_{i,(N-1)}}
\end{bmatrix}$$

- Converting from serial to parallel vector form:

$$\tilde{y} = D_{\alpha \Omega} C_p W^* \tilde{s} + \tilde{\eta}$$

- An issue, the transfer matrix (in brackets) is no longer diagonal, but diagonally dominant

$$\tilde{s} = [WC_{pr} D_{\alpha \Omega} C_p W^*] \tilde{s} + \tilde{\eta}$$
The Problem – a time & frequency dispersive channel with a particularly large delay-Doppler product

- In typical wireless applications frequency dispersion is a fraction of a tone spacing.
- In the problem considered here, frequency dispersion greatly exceeds the tone spacing
Prior work related to the use of OFDM with frequency selective time-varying fading channels:

  - Propose a MMSE-based OFDM receiver structure
  - Estimate channel by curve fitting over time-frequency using pilots, based on statistical model of time-varying impulse response

- Linnartz, Gorhkov, *Doppler-resistant OFDM receivers for Mobile Multimedia Communications*, PIMRC 2000, London, Sept 2000, and

- Mostofi, Cox, *ICI Mitigation for Pilot-Aided OFDM Mobile Systems*:
  - MMSE ICI receiver based on piece-wise linear approximation to channel response at each tone
    - Suitable for limited Doppler spread

- Y. Li, L.J. Cimini, and N.R. Sollenberger, *Robust channel estimation for OFDM systems with rapid dispersive fading channels*, IEEE, Tr. on Comm. vol. 46, pp. 902-915, July 1998:

- Many others
An MMSE-based OFDM receiver structure is developed for the general, deterministically modeled delay-Doppler channel.

- The received signal vector $\tilde{r}$ through the delay-Doppler channel $D_{\Omega}$:
  $$\tilde{r} = C_{pr} D_{\Omega} C_p W^* s + \eta$$
  $$\tilde{r} = F s + \eta$$
  $$F = C_{pr} D_{\Omega} C_p W^*$$

- The MMSE solution is given by:
  $$\hat{s} = (\hat{F}'\hat{F} + I\sigma_n^2)^{-1}\hat{F}'\tilde{r}$$
  requiring estimate of delay-Doppler channel
  $$\hat{F} = C_{pr} \hat{D}_{\Omega} C_p W^*$$

- Comparing delay-Doppler MMSE receiver to delay-only OFDM receiver:
  - OFDM - NxN FFT  \[ \hat{s} = \hat{H}^* W \tilde{r} \]
  - MMSE - $N_s \times N_s$ inverse  \[ \hat{s} = (\hat{F}'\hat{F} + I\sigma_n^2)^{-1}\hat{F}'\tilde{r} \]
  - where $N_s$ – number of symbols; $N$ – FFT size
Simulation results

Comparison of OFDM performance in delay-only case to that of MMSE in delay-Doppler case
Simulations comparing the OFDM receiver in the delay-only case to that of the MMSE receiver through the same channel but with a random Doppler added to each channel tap

Parameter Name | Value
--- | ---
FFT Size = $N_{\text{FFT}}$ | 1024
Tone Spacing -- (rad) -- $2\pi/N_{\text{FFT}}$ | 0.0061
Cyclic Prefix | 160 samples
Cyclic Suffix | 6 samples
No. of bits per packet | 99
Channels evaluated -- 11 tap channel with Doppler spread of a few tones, and an several hundred tap channel spanning the entire 160 sample cyclic extension with a Doppler spread of over 500 tones.

11 tap channel

- Tap weights
- Tap Dopplers (rad/sample)

many tap channel

- Tap weights
- Tap Dopplers (rad/sample)

- Frequency responses computed for the case in which Doppler = 0

• Note: Frequency responses computed for the case in which Doppler = 0
Performance of OFDM and MMSE Receivers with perfect channel knowledge

- Performance of MMSE receiver in delay-Doppler channel closely matches that of OFDM in delay-only channel
- Performance lost to Doppler recovered with MMSE receiver, but at a significant increase to computational expense

Bit Error Rate vs Eb/No - in AWGN, Delay-only, and Delay-Doppler Channels

AWGN

11 tap delay-only & delay-Doppler channels

Many-tap delay-only & delay-Doppler channels
Parameter Estimation

An estimation approach for a sensor array channel structure
We consider the time-varying multi-path channel involving $K$ rays from one transmitter to a P-node array.

- Each ray involves an unknown gain $\alpha_k$, fractional delay $N_k$, and Doppler shift $\theta_k$ to be estimated.
- Array signal processing parameters, $\beta_{mp}, M_{mp}, \omega_{mp}$, are known.
- For this channel we construct the $F$ matrix relating $\vec{S}$ to $\vec{R}$.
The $F$ matrix for the case of $K$ rays

$$F_k (N_k, \Omega_k, \beta_{mk}, M_{mk}, \omega_{mk}) = C_{pr} \sum_{m=1}^{M} \left[ (\beta_{mk} e^{-j\Omega_k M_{mk}}) D_\tau (N_k + M_{mk}) D_\Omega (\Omega_k + \omega_{mk}) \right] C_p W^*$$

$$F(\alpha_k, F_k) = \sum_{k=1}^{K} \alpha_k F_k$$

$$\vec{r} = F\vec{s} + \eta$$

Where the channel Dopper & delay matrix operators are:

$$D_\Omega (\Omega) = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & e^{j\Omega (1)} & 0 & \cdots \\
\vdots & 0 & \ddots & 0 \\
0 & \cdots & 0 & e^{j\Omega (N-1)}
\end{bmatrix}$$

$$D_\tau (\tau) = W^* \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & e^{-j\tau_1} & 0 & \cdots \\
\vdots & 0 & \ddots & 0 \\
0 & \cdots & 0 & e^{-j\tau (N-1)}
\end{bmatrix} W$$

- Note that $F$ is a function of the known array processing parameters, $\beta_{mp}, M_{mp}, \omega_{mp}$
- and the unknown ray parameters $\alpha_k, \theta_k, N_k$

- Requiring estimation of 4 real parameters per ray
• Training symbol vectors made entirely of pilots are transmitted
  – Series of sample-domain vectors $\tilde{r}$ used for parameter estimation

• Channel Doppler, delay, and gain estimates generated sequentially, not jointly
  – Estimates of Doppler generated via ESPRIT, independent and without knowledge of channel
    delay and gain estimates
  – Delay estimates generated via series of 1-D searches, also pairing Dopplers to particular rays
  – $\hat{F}_k$ matrices computed, and channel gains estimated as final step

MMSE receiver and parameter estimation architecture showing separate estimation of
Doppler, delay, and channel gain
Use of ESPRIT with Multiple Successive Training OFDM Symbol Blocks to Achieve Separable Estimation of Residual Dopplers and Delays Associated with Multiple MBAJ Channels input to MBAJ

- **Estimation Doppler via ESPRIT:**
  - train with \(K+1\) repeated OFDM training symbols
  - estimates are the eigenvalues of a matrix
    - avoids a multi-dimensional search and is invariant to unknown gains and time delays

- **Subsequent estimation of residual time delays**
  - using Doppler shift estimates
  - uses eigenvector information from ESPRIT to decouple superimposed rays allowing for separable estimation of residual time delays (pairing problem addressed)

- **Unknown channel gains subsequently estimated via Least Square Error solution**

- **Total** \(F\) matrix compute given estimates and used over subsequent OFDM Frames for symbol data detection
ESPRIT Based Estimation of Delay – consider this example involving 3 rays:

\[ z_1 = \alpha_1 \left[ C_{pr} \left( \sum_{1,i} \beta_{1i} D_{\tau,1i} D_{\omega,1i} \right) \right] D_{\tau_1}^{\text{residual}} D_{\omega_1}^{\text{residual}} (C_p W^* s) \]

\[ z_2 = \alpha_2 \left[ C_{pr} \left( \sum_{2,i} \beta_{2i} D_{\tau,2i} D_{\omega,2i} \right) \right] D_{\tau_2}^{\text{residual}} D_{\omega_2}^{\text{residual}} (C_p W^* s) \]

\[ z_3 = \alpha_3 \left[ C_{pr} \left( \sum_{3,i} \beta_{3i} D_{\tau,3i} D_{\omega,3i} \right) \right] D_{\tau_3}^{\text{residual}} D_{\omega_3}^{\text{residual}} (C_p W^* s) \]

Send the same OFDM training symbol for 4 successive blocks to generate the following output vectors:

\[ y_0 = z_1 + z_2 + z_3 \]

\[ y_1 = e^{j(N+L)\omega_1} z_1 + e^{j(N+L)\omega_2} z_2 + e^{j(N+L)\omega_3} z_3 \]

\[ y_2 = e^{j2(N+L)\omega_1} z_1 + e^{j2(N+L)\omega_2} z_2 + e^{j2(N+L)\omega_3} z_3 \]

\[ y_3 = e^{j3(N+L)\omega_1} z_1 + e^{j3(N+L)\omega_2} z_2 + e^{j3(N+L)\omega_3} z_3 \]
Cast these equations as an eigenvalue problem, thereby avoiding a multi-dimensional search to find Doppler values

- With quantities defined as on previous slide, it follows that:

\[
G_1 = \begin{bmatrix} y_0 & y_1 & y_2 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} \Delta \quad G_2 = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} \Phi \Delta
\]

where

\[
\Phi = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{pmatrix} \quad \phi_1 = e^{j(N+L)\omega_1} \quad \phi_2 = e^{j(N+L)\omega_2} \quad \phi_3 = e^{j(N+L)\omega_3}
\]

Thus we can write: \( G_1 \Psi = G_2 \) allowing \( \Psi \) to be determined numerically

The solution is \( \Psi = \Delta^{-1} \Phi \Delta \) implying that

\[
\phi_1 = e^{j(N+L)\omega_1} \quad \phi_2 = e^{j(N+L)\omega_2} \quad \& \quad \phi_3 = e^{j(N+L)\omega_3}
\] are eigenvalues of \( \Psi \)
Here is an example involving a 3 ray channel with unknown delays, Doppler, and gains -- the 3 Doppler values are estimated correctly.

\[
\phi_3 = e^{j(N+L)\omega_3} \\
\phi_1 = e^{j(N+L)\omega_1} \\
\phi_2 = e^{j(N+L)\omega_2}
\]
Use eigenvector information from ESPRIT to separate the three channel contributions summed at the array output:

\[
G_1 = \begin{bmatrix} y_0 & y_1 & y_3 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} \Delta \\
G_2 = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} \Phi \Delta
\]

\[
\Phi = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{pmatrix} \quad \phi_1 = e^{j(N+L)\omega_1} \\
\phi_2 = e^{j(N+L)\omega_2} \\
\phi_3 = e^{j(N+L)\omega_3}
\]

\[
\phi_i = e^{j(N+L)\omega_i}, \text{ } i = 1, 2, 3 \text{ are eigenvalues of solution to } G_1 \Psi = G_2
\]

\[
\Psi = E \Lambda E^{-1} \quad \text{where} \quad \Lambda = \Phi \quad \text{and} \quad E = \Delta^{-1} DP
\]

where \( P \) is a permutation matrix and \( D \) is a diagonal matrix representing unknown scalings, since eigenvectors are only unique to within a multiplicative scalar

THUS: eigenvectors of \( \Psi \) decouple superposition of sensor array output contributions from channel 1, channel 2 & channel 3:

\[
G_1 E = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} DP
\]
Pairing problem and residual delay estimation:

\[ G_1 E = [\zeta_1 | \zeta_2 | \zeta_3] = [z_1 | z_2 | z_3]DP \]

\[ \tilde{z}_1 = \left[ C_{pr} \left( \sum_{1,i} \beta_{1i} D_{\tau,1i} D_{\omega,1i} \right) \right] D_{\tau_1}^{\text{residual}} D_{\omega_1}^{\text{residual}} \left( C_p W^* s \right) \]

\[ \tilde{z}_2 = \left[ C_{pr} \left( \sum_{2,i} \beta_{2i} D_{\tau,2i} D_{\omega,2i} \right) \right] D_{\tau_2}^{\text{residual}} D_{\omega_2}^{\text{residual}} \left( C_p W^* s \right) \]

\[ \tilde{z}_3 = \left[ C_{pr} \left( \sum_{3,i} \beta_{3i} D_{\tau,3i} D_{\omega,3i} \right) \right] D_{\tau_3}^{\text{residual}} D_{\omega_3}^{\text{residual}} \left( C_p W^* s \right) \]

There is an unknown gain on each channel -- absorb into unknown diagonal scaling matrix \( D \)

Combinatoric pairing problem: six possibilities to assess in case of 3 channels feeding into sensor array
For each of the permuted pairings, search for the delays producing a match (to within a scalar) of the permuted eigenvector and the \( \tilde{z} \) vectors

\[
G_1 E = \begin{bmatrix}
\zeta^{(1)} & \zeta^{(2)} & \zeta^{(3)}
\end{bmatrix}
\]

\((\delta, \varepsilon, \gamma) \in \{(1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), (3,2,1)\}\)

\[
\tilde{z}_1(\omega^{(\delta)}, \hat{\tau}_1) = \left[ C_{pr} \left( \sum_{1,i} \beta_{1i} D_{\tau,1i} D_{\omega,1i} \right) \right] D_{\hat{\tau}_1}^{\text{residual}} D_{\hat{\omega}^{(\delta)}}^{\text{residual}} (C_p W^* s)
\]

\[
\tilde{z}_2(\omega^{(\varepsilon)}, \hat{\tau}_2) = \left[ C_{pr} \left( \sum_{2,i} \beta_{2i} D_{\tau,2i} D_{\omega,2i} \right) \right] D_{\hat{\tau}_2}^{\text{residual}} D_{\hat{\omega}^{(\varepsilon)}}^{\text{residual}} (C_p W^* s)
\]

\[
\tilde{z}_3(\omega^{(\gamma)}, \hat{\tau}_3) = \left[ C_{pr} \left( \sum_{3,i} \beta_{3i} D_{\tau,3i} D_{\omega,3i} \right) \right] D_{\hat{\tau}_3}^{\text{residual}} D_{\hat{\omega}^{(\gamma)}}^{\text{residual}} (C_p W^* s)
\]

\[
\Gamma(\delta, \varepsilon, \gamma) = \begin{cases}
\text{minimum} & \begin{cases}
0 < \hat{\tau}_1 < LT_s
\end{cases}
\begin{cases}
\text{smallest singular value of}
\begin{bmatrix}
\zeta^{(\delta)} & \tilde{z}(\omega^{(\delta)}, \hat{\tau}_1)
\end{bmatrix}
\end{cases}
\end{cases}
\end{cases}
\]

\[
+ \begin{cases}
\text{minimum} & \begin{cases}
0 < \hat{\tau}_2 < LT_s
\end{cases}
\begin{cases}
\text{smallest singular value of}
\begin{bmatrix}
\zeta^{(\varepsilon)} & \tilde{z}(\omega^{(\varepsilon)}, \hat{\tau}_2)
\end{bmatrix}
\end{cases}
\end{cases}
\]

\[
+ \begin{cases}
\text{minimum} & \begin{cases}
0 < \hat{\tau}_3 < LT_s
\end{cases}
\begin{cases}
\text{smallest singular value of}
\begin{bmatrix}
\zeta^{(\gamma)} & \tilde{z}(\omega^{(\gamma)}, \hat{\tau}_3)
\end{bmatrix}
\end{cases}
\end{cases}
\]
ESPRIT Based Method for Estimating Residual Dopplers for 3 Channels Feeding into array with Delay-Doppler Taps: Step 2. Pairing no. 3 (correct)
Channel gain estimation is accomplished as a final step after estimation of individual $F_k$:

- Recall that array processor output:
  \[
  \tilde{r} = F\bar{s} + \eta
  \]

- Transmission of a train symbol vector $\bar{s}_p$ allows this equation to be written:
  \[
  \tilde{r} = \left[ \sum_{k=1}^{K} \alpha_k \hat{F}_k \right] \bar{s}_p + \eta = \left[ \hat{F}_1 \bar{s}_p \quad \hat{F}_2 \bar{s}_p \quad \ldots \quad \hat{F}_K \bar{s}_p \right] \bar{\alpha} = F_S \bar{\alpha}
  \]

- The least-square solution for alpha:
  \[
  \hat{\alpha} = \left[ (F_S' F_S)^{-1} F_S' \right] \tilde{r}
  \]
Conclusions

• An MMSE-based OFDM receiver robust to large Doppler spread was developed
  – No restriction on magnitude of Doppler
  – BER Performance approaches that of OFDM receiver for same channel (less Doppler)

• A method for separately estimating channel gains, time delays and Doppler developed and demonstrated
  – dimensionality and complexity lower than a joint estimation technique

• Current Activity:
  – Evaluating sensitivity of MMSE-based OFDM detection performance to estimation errors
  – Evaluating sensitivity of channel estimates to noise
  – Investigating estimation and receiver performance with non-constant channel coefficients associated with transmitter motion