MIMO Radar Application to Moving Target Detection in Homogenous Clutter

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Outline

- Motivation
- Signal models
- Moving target detectors
- Examples
- Conclusions
Radar Architectures

Key features of MIMO radar:
- Multiple transmitters, independent waveforms
- Statistical MIMO radar
- Widely distributed sensors
- Centralized processing
- RCS diversity of targets
Problem Formulation

- Ground radar application: detection of moving target with interference from ground clutter
- Single antenna/phased array radars hindered by targets with blind or low radial velocities
- Can performance improve with distributed sensors and centralized processing?

**Spatial diversity with respect to target movement**
Assumptions

- Transmit and receive apertures are stationary.
- Clutter is homogenous; its return is Gaussian distributed, and the temporal correlation is known or estimated independently.
- During observation time, target does neither “leave” the cell under test nor alter aspect (RCS).
- Transmit and receive elements in MIMO and multistatic radar are sufficiently separated to observe uncorrelated RCS’s.
- Signal model accounts only for small-scale effects.
- Receiving sensor have acquired the time reference of the transmitting sensors.
- We formulate a detection problem for a target with unknown amplitude and unknown velocity vector.
Resolution Cell in Phased Array

- Resolution cell defined in range and azimuth

- Size of resolution cell:
  - Range: bandwidth of transmitted waveform
  - Azimuth: aperture size
Resolution Cell in MIMO Radar

- Target located at the intersection of ellipses representing the loci of constant time delays

- Size of resolution cell determined by the radar signal bandwidth

- Stationary target: the optimal detector that exploits the diversity of RCS processes the sensor outputs non-coherently [Fishler et al. 2006]

- Extend these results to moving target
Signal Model

- Let cell under test (CUT) at $\vec{X}$;
  transmit elements at $\vec{T}_k$ for $k = \{1, \ldots, M\}$
  and receive elements at $\vec{R}_l$ for $l = \{1, \ldots, N\}$
  K pulses transmitted at PRI intervals

- Filter matched to transmitted waveform and
  CUT for the n-th pulse:
  \[
  y_{l,k,n}(\vec{X}) = \int r_{l}^*(t - kT_{PRI})s_x(t - nT_{PRI} - \tau_{tk}(\vec{X}) - \tau_{nl}(\vec{X}))dt
  \]

- Form vector of K observations
  \[
  y_{l,k}(\vec{X}) = \begin{cases} 
  h_{l,k} d_{l,k}(v_x, v_y) + c_{l,k} + n_{l,k} & H_1 \\
  c_{l,k} + n_{l,k} & H_0
  \end{cases}
  \]
• Bistatic target complex amplitude $h_{l,k}$

• AWGN $E \left\{ n_{l,k} n_{l,k}^H \right\} = \sigma_w^2 I_{K \times K}$

• Clutter spatially white $E \left\{ c_{l,k} c_{l',k'}^H \right\} = 0$

• Clutter temporally corelated $E \left\{ c_{l,k} c_{l,k}^H \right\} = C'$

• Placing the cell under test at the origin, the transmit elements at $\theta_k$, and the receive elements at $\theta_l$,

$$f_{l,k} = (\cos \theta_k + \cos \theta_l) \frac{v_x}{\lambda} + (\sin \theta_k + \sin \theta_l) \frac{v_y}{\lambda}$$

• Target Doppler vector

$$d_{l,k}(v_x, v_y) = \left[ 1, e^{j2\pi f_{l,k} T_{PRI}}, \ldots, e^{j2\pi f_{l,k} T_{PRI} (K-1)} \right]^T$$
MIMO Radar MTD Detector

- Conditional distributions under the two hypotheses applied to CUT $\bar{X}$

$$p\left(y(\bar{X})\mid\{h_{l,k}\},v_x,v_y,H_1\right) = \prod_{k=1}^{M} \prod_{l=1}^{N} e^{-\left(y_{l,k}(\bar{X})-h_{l,k}d_{l,k}(v_x,v_y)\right)^H \mathbf{C}^{-1} \left(y_{l,k}(\bar{X})-h_{l,k}d_{l,k}(v_x,v_y)\right)}$$

$$p\left(y(\bar{X})\mid H_0\right) = \prod_{k=1}^{M} \prod_{l=1}^{N} e^{-y_{l,k}(\bar{X})^H \mathbf{C}^{-1} y_{l,k}(\bar{X})}$$

- Unknown parameters: $\{h_{l,k}\},v_x,v_y$

- GLRT test statistic for MTD target detection

$$\zeta = \max_{v_x,v_y} \sum_{k=1}^{M} \sum_{l=1}^{N} \frac{\left|d_{l,k}(v_x,v_y)^H \mathbf{C}^{-1} y_{l,k}(\bar{X})\right|^2}{d_{l,k}(v_x,v_y)^H \mathbf{C}^{-1} d_{l,k}(v_x,v_y)} \begin{cases} H_1 & > \gamma \\ H_0 & < \gamma \end{cases}$$

- Maximization with respect to $(v_x,v_y)$ pursued numerically
Multistatic, Distributed Processing

- The MIMO MTD GLRT detector features:
  - Centralized processing
  - Sensors need to maintain only local phase reference: non-coherent processing between sensors

- Compare MIMO MTD to multistatic and phased array

- Consider a multistatic system that performs the GLRT in distributed fashion: each sensor performs its own optimization and transmits the outcome to the central processor

\[
\xi = \sum_{k=1}^{M} \sum_{l=1}^{N} \max_{v_{x,l,k}, v_{y,l,k}} \frac{\left| d_l(v_{x,l,k}, v_{y,l,k})^H C^{-1} y_{l,k}(\hat{X}) \right|^2}{d_l(v_{x,l,k}, v_{y,l,k})^H C^{-1} d_l(v_{x,l,k}, v_{y,l,k})} \begin{cases} > \gamma & H_1 \\ < \gamma & H_0 \end{cases}
\]
MTD, Phased Array

- With distributed processing, the chances for false alarms increase since individual sensors are not constrained to the same \((v_x, v_y)\) estimates.

- Phased array: space-time coherent processing

\[
\xi = \max_{v_{x,l,k}, v_{y,l,k}} \frac{|d(v_{x,l,k}, v_{y,l,k})^H C^{-1} y(\bar{X})|^2}{d(v_{x,l,k}, v_{y,l,k})^H C^{-1} d(v_{x,l,k}, v_{y,l,k})} \begin{cases} H_1 \quad \text{if} \quad \gamma > \gamma_0 \\ H_0 \quad \text{if} \quad \gamma < \gamma_0 \end{cases}
\]

- Under \(H_1\), a phased array observes only a single Doppler shift. Performance degrades for small radial target velocities.

- Cannot exploit target RCS fluctuations.
Missed Detections

- With single antenna or phased arrays, detections are missed when the target radial velocity falls in the frequency band notched out by the clutter canceler.

- For a velocities uniform distributed in $[0, \pi]$:

$$\Pr\{\text{miss}\} = \Pr\left\{ \left| \sin \varphi \right| \leq \frac{\lambda B_{\text{clutter}}}{2 |\vec{v}|} \right\} \approx \frac{1}{\pi} \frac{\lambda B_{\text{clutter}}}{|\vec{v}|}$$

- Intuitively, missed detection can be avoided by having sufficient number of sensors such that $\varphi$ is never too small.
How Can MIMO Help MTD?

Observed Doppler shift at sensor $l$ due to transmission from sensor $k$

$$f_{l,k} = (\cos \theta_k + \cos \theta_l) \frac{v_x}{\lambda} + (\sin \theta_k + \sin \theta_l) \frac{v_y}{\lambda}$$

$$= 2 \frac{\vec{v}}{\lambda} \cos(\varphi') \cos\left(\frac{\theta_k - \theta_l}{2}\right)$$

- $\varphi'$: is the angle between the target velocity vector and the bisector of the transmitter-target-receiver angle

- For a given target velocity, the larger the Doppler shift, the better the chance of target not affected by the clutter notching filter

- Transmitters increase the Doppler shift
Design Considerations

\[ f_{l,k} = 2 \frac{|v|}{\lambda} \cos(\phi') \cos\left(\frac{\theta_k - \theta_l}{2}\right) \]

Design considerations:

- Distribute sensors over large azimuth sector: some sensor will yield desired \( \cos \phi' \approx 1 \)

- To get large Doppler shifts, keep \( (\theta_k - \theta_l) \) small by using sensors as both transmitters and receivers
Simulation Settings

Pulse Repetition Frequency \( \text{PRF} = 2 \text{ kHz} \)
Carrier frequency \( 1 \text{ GHz} \)
Number of coherent pulses \( K = 10 \)
Clutter-to-noise ratio \( 30 \text{ dB} \)
Clutter PSD \( \text{Gaussian} \)
Clutter RMS spread \( 1.25 \text{ m/s} \)
Target velocity \( 300 \text{ km/h} \)

Transmit and receive elements:

Receive elements:
\[
\begin{align*}
\theta_{l,1x1} &= \{0^\circ\} \\
\theta_{l,1x8} &= \{0^\circ, 13^\circ, 26^\circ, 38^\circ, 50^\circ, 62^\circ, 75^\circ, 90^\circ\} \\
\theta_{l,1x4} &= \theta_{l,2x4} = \{0^\circ, 30^\circ, 60^\circ, 90^\circ\}
\end{align*}
\]

Transmit elements:
\[
\begin{align*}
\theta_{k,1x1} &= \{0^\circ\} \\
\theta_{k,1x4} &= \theta_{k,1x8} = \{5^\circ\} \\
\theta_{k,2x4} &= \{5^\circ, 85^\circ\}
\end{align*}
\]
False Alarms Example

- Empirical cdf of test statistic under $H_0$
  - MIMO, centralized processing
  - Multistatic, distributed

- Distributed multistatic processing results in higher false alarms
Constant RCS Example

- Empirical cdf of test statistic when target present ($H_1$)
- Captures effects:
  - Random approach angle uniformly distributed
  - Clutter cancellation
  - AWGN noise
- Interpretation: fraction of test statistics that have lower value than abscissa. If threshold set at abscissa, ordinate measures $P_{\text{miss}}$.
- MIMO 2x4:
  - Superior for threshold setting such that $P_{\text{miss}} < 0.05$
  - Most stable statistics
More on the RCS Example

- In the previous example, transmitted power was normalized by the number of transmit antennas.

- More realistic for radar: power increases linearly with the number of transmit antennas.

- Now MIMO 2x4 outperforms MIMO 1x8 for any threshold setting.
Fluctuating RCS Example

- Fluctuating RCS; sensors experience independent RCS returns

- Captures effects:
  - Random (exponential) target RCS
  - Random approach angle uniformly distributed
  - Clutter cancellation
  - AWGN noise

- $P_{\text{miss}}$ increases due to target fading

- Increasing the number of sensors not only improves performance of MTD, but also serves to provide diversity against target fading
ROC’s

Constant RCS

Fluctuating RCS
Conclusions

- Focused on a MIMO radar architecture featuring:
  - Widely distributed sensors
  - Centralized processing
  - RCS diversity
- Investigated a scenario of ground based radar and a moving target
- Developed the GLRT for a moving target for: MIMO radar, a multistatic radar with distributed processing, phased array
- The GLRT processor combines the sensor outputs non-coherently
- MIMO radar advantages for MTD:
  - Diversity of aspects mitigates against low radial speed
  - Multiple transmitters contribute to increase Doppler shifts
- Presented examples demonstrating the performance of various radar architectures
- Ongoing work: high resolution target location using a coherent mode for MIMO radar
Resolving Multiple Scatterers

High resolution processing of 4 targets

**Coherent processing**

**Noncoherent processing**