ADAPTIVE RADAR DETECTION FOR
EXTENDED AND DISTRIBUTED TARGETS
WITHOUT ASSIGNMENT OF SECONDARY DATA

by

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**Problem Formulation**

Assume that:

- an array of \( N \) sensors (\( N_a \) antennas times \( N_t \) pulses) collects returns from \( K \) consecutive range cells;
- a distinct set of secondary data (i.e., data free of signal components) is **not available**, but a certain number of range cells, not a priori known, contain noise only;
- \( H \) cells at most may contain target’s or targets’ scatterers with \( K - H \geq N \).

Denote by

- \( r_k \) the \( N \)-dimensional vector containing returns from the \( k \)-th cell;
- \( n_k \) the corresponding noise vector: the \( n_k \)'s are independent, zero mean, complex normal random vectors.

The detection problem at hand is the following

\[
\begin{align*}
\mathcal{H}_0 &: r_k = n_k, \quad k \in \Omega, \\
\mathcal{H}_1 &: \begin{cases}
  r_k = \alpha_k s + n_k, & k \in \Omega_T, \\
  r_k = n_k, & k \in \Omega \setminus \Omega_T,
\end{cases}
\end{align*}
\]

where

- \( \Omega \) denotes the set of range cells under test;
- \( \Omega \setminus \Omega_T \) denotes the **unknown** set of range cells free of signal components;
- \( s \) is the known steering vector;
- the \( \alpha_k \)'s, \( k \in \Omega_T \), are the **unknown** amplitudes of the target’s (targets’) scattering centers;
- the covariance matrix of the noise \( M \) is **unknown**.
Detection of Extended Targets

Suppose that $\Omega_T$ is a subset of $\Omega$ formed by $H$ consecutive cells ($K - H \geq N$).

The plain GLRT for extended target, which requires maximization with respect to $M$ of the likelihood function under the $H_0$ hypothesis and with respect to $M$, the $\alpha_k$'s, and $\Omega_T$ of the likelihood function under the $H_1$ hypothesis, is given by

$$\max_{\Omega_T} \frac{\det(R_0 + S)}{\det(R_1 + S)} \begin{cases} \frac{H_1}{H_0} \geq \gamma \\ \frac{H_1}{H_0} < \gamma \end{cases}$$ (1)

where

$$S = \sum_{k \in \Omega \setminus \Omega_T} r_k r_k^\dagger,$$ (2)

$$R_0 = \sum_{k \in \Omega_T} r_k r_k^\dagger,$$

and

$$R_1 = \sum_{k \in \Omega_T} \left( r_k - \frac{s^\dagger S^{-1} r_k}{s^\dagger S^{-1} s} s \right) \left( r_k - \frac{s^\dagger S^{-1} r_k}{s^\dagger S^{-1} s} s \right)^\dagger.$$

As to $\gamma$, it is the detection threshold to be set according to the preassigned Probability of False Alarm ($P_{fa}$).
Detection of multiple point-like targets

Detection of multiple point-like targets (and, hence, of extended ones as a special case) can be performed by resorting to the two-step GLRT-based design procedure:

- first assume that the covariance matrix of the disturbance $M$, or its structure $\Sigma$, is known and implement the GLRT fed by the $K$ range cells and assuming that all of the cells contain a target scatterer (i.e., $\Omega_T = \Omega$);
- then, replace the unknown matrix ($M$ or $\Sigma$, respectively) with a proper estimate, $\hat{\Sigma}$ say.

It is not difficult to show that the above design procedure yields

$$\sum_{k \in \Omega} \frac{|s^\dagger \hat{\Sigma}^{-1} r_k|^2}{s^\dagger \hat{\Sigma}^{-1} s} \begin{cases} H_1 \quad \text{if } H_{1i} \geq \gamma \quad \text{for the case that we assume } M \text{ known} \\ H_0 \end{cases}$$

for the case that we assume $M$ known and

$$\sum_{k \in \Omega} \frac{|s^\dagger \hat{\Sigma}^{-1} r_k|^2}{s^\dagger \hat{\Sigma}^{-1} s \sum_{k \in \Omega} r_k^\dagger \hat{\Sigma}^{-1} r_k} \begin{cases} H_1 \quad \text{if } H_{1i} \geq \gamma \\ H_0 \end{cases}$$

for the case that we assume $\Sigma$ known (in the first step of the design procedure).
Detection of multiple point-like targets (cont.d)

The two-step design procedure can be modified to incorporate a priori knowledge about the maximum number $H$ of point-like targets (or the maximum target extension for an extended one):

- first assume that $M$ or $\Sigma$ is known and implement the GLRT over the $K$ range cells under test, but maximizing the likelihood function under the $H_1$ hypothesis also with respect to $\Omega_T$, with $\Omega_T \subset \Omega$;
- then, replace the unknown matrix ($M$ or $\Sigma$, respectively) with a proper estimate $\hat{\Sigma}$.

This design procedure yields

$$
\sum_{k \in \hat{\Omega}_T} \frac{|s|\hat{\Sigma}^{-1}r_k|^2}{s^T\hat{\Sigma}^{-1}s} \frac{H_1}{H_0} > \gamma
$$

for the case that we assume $M$ known in the first step of the design procedure and

$$
\sum_{k \in \hat{\Omega}_T} \frac{|s^T\hat{\Sigma}^{-1}r_k|^2}{s^T\hat{\Sigma}^{-1}s} \frac{H_1}{H_0} > \gamma
$$

otherwise.

As to $\hat{\Omega}_T$, it denotes the set of integers indexing the range cells in $\Omega$ which correspond to the $H$ greatest values of

$$
\frac{|s^T\Sigma^{-1}r_k|^2}{s^T\Sigma^{-1}s}, \quad k \in \Omega,
$$

and

$$
\frac{|s^T\Sigma^{-1}r_k|^2}{s^T\Sigma^{-1}s} \sum_{k \in \Omega} r_k \Sigma^{-1}r_k, \quad k \in \Omega,
$$

respectively.

**Note that:** for an extended target the maximizer $\hat{\Omega}_T$ might be computed under the constraint that the range cells indexed by $\Omega_T$ are $H$ consecutive ones.
Detection of multiple point-like targets (cont.d)

**Estimation of $\Sigma$ or $M$:** we propose to resort to either the normalized sample covariance matrix

$$\hat{\Sigma} = \frac{N}{K} \sum_{k \in \Omega} r_k r_k^\dagger$$


**Note that:** both estimators should guarantee reliable estimates of $\Sigma$ when fed by a random sample contaminated by targets’ scatterers.

It is important to stress that coupling:

- detectors (4) and (6) with the normalized sample covariance matrix guarantees the CFAR property with respect to the noise power $\sigma^2$;

- detectors (4) and (6) with the recursive estimate of $\Sigma$ guarantees the CFAR property with respect to both $\sigma^2$ and $\Sigma$, even at the price of a certain additional performance loss.
Performance Assessment

We resort to standard Monte Carlo counting techniques to evaluate the threshold necessary to ensure a preassigned value of Probability of False Alarm ($P_{fa}$) and of Probability of Detection ($P_d$).

The simulated scenarios assume:

- $N = 8$ sensors, $K = 39$ range cells, $H = 8$, and four scatterers;
- the $N$-dimensional space-time steering vector is defined as
  \[ s = [1 \cdots 1]^T; \]
- the noise is an exponentially-correlated disturbance with one-lag correlation coefficient $\rho = 0.95$;
- the SNR is defined as
  \[ \text{SNR} = \frac{\sum_{i \in \Psi} |\alpha_i|^2 \|s\|^2}{N\sigma^2} \]
  where $\Psi \subset \Omega$ is a set of integers indexing the actual range cells which, under the $H_1$ hypothesis, contain the scatterers.
Performance assessment: extended target

Figure 1: $P_d$ vs SNR for detector (3) (dashed line), detector (4) (dashed line with point marker), detector (5) (solid line), and detector (6) (solid line with point marker), all coupled with the normalized sample covariance matrix.
Performance assessment: extended target (cont.d)

In order to assess the sensitivity of the non-CFAR detector (5) and of the MGLRT proposed by Gerlach, with respect to possible mismatches between the nominal and the actual value of the noise power $\sigma^2$, we have estimated the $P_{fa}$ for several values of $\sigma^2$, by using the threshold $\gamma$ which guarantees a nominal $P_{fa}$ of $10^{-2}$ with $\sigma^2 = 2$. Results are summarized in Table 1.

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<th>$\sigma^2$</th>
<th>Detector (5)</th>
<th>MGLRT</th>
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<tr>
<td>2</td>
<td>0.0120</td>
<td>0.0108</td>
</tr>
<tr>
<td>3</td>
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<td>4</td>
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<td>0.9532</td>
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<tr>
<td>5</td>
<td>0.9802</td>
<td>0.9980</td>
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Table 1: Estimated values of $P_{fa}$ of detector (5) and of the MGLRT in presence of mismatch between the actual $\sigma^2$ and the design one ($\sigma^2 = 2$). The nominal value of the $P_{fa}$ is $10^{-2}$. 
Performance assessment: extended target (cont.d)

Figure 2: $P_d$ vs SNR for detector (5) (no marker) and detector (6) (point marker), both coupled with the normalized sample covariance matrix; MGLRT detector (cross marker); detector (1) (triangle marker).
Performance assessment: multiple point-like targets

Figure 3: $P_d$ vs SNR for detector (3) (dashed line), detector (4) (dashed line with point marker), detector (5) (solid line), and detector (6) (solid line with point marker), coupled with the normalized sample covariance matrix; detector (3) (dashed line with square marker), detector (4) (dashed line with circle marker), detector (5) (solid line with square marker), and detector (6) (solid line with circle marker), coupled with the recursive estimator.
Performance assessment: multiple point-like targets (cont.d)

Figure 4: $P_d$ vs SNR for detector (5) (no marker) and detector (6) (point marker), both coupled with the normalized sample covariance matrix; MGLRT detector (cross marker); detector (5) (square marker) and detector (6) (circle marker), both coupled with the recursive estimator.
Conclusions

- We have addressed adaptive detection of extended and multiple point-like targets, embedded in Gaussian disturbance with unknown statistics, without assignment of distinct secondary data.

- We have proposed one-step and two-step GLRT-based procedures with the true covariance matrix (or its structure) replaced by the normalized sample covariance matrix or the recursive estimator proposed by Conte et al. in [“Recursive Estimation of the Covariance Matrix of a Compound-Gaussian Process and Its Application to Adaptive CFAR Detection,” IEEE Trans. on Signal Processing, Vol. 50, No. 8, pp. 1908-1915, August 2002.]

- Remarkably, detector (1) has the CFAR property with respect to both the structure of the covariance matrix and the noise power, while detectors (4) and (6) guarantee the CFAR property with respect to the power level only, when coupled with the normalized sample covariance matrix, and with respect to the power and the structure, when coupled with the recursive estimate.

- A preliminary performance assessment seems to indicate that detector (6), coupled with the normalized sample covariance matrix, is a viable means to detect extended and multiple point-like targets in uncertain scenarios.