Case of a Short Data Record for Both Training and Signal Detection

Brian E Freburger
Naval Air Warfare Center

Donald W. Tufts
University of Rhode Island
Comparison Structure

All Comparisons are for the GSLC Structure
Cross Spectral Metric

Perform an SVD on the Covariance and Select the p Singular Vectors Based on the Cross Spectral Metrics

\[ W_{CSM} = U_p \Sigma^{-2} U^H p r_d Z \]

where the CSM’s are given by \[ \frac{|u_j^H r_d Z|^2}{\sigma_j^2} \] for \( j = 1 \ldots N - 1 \)

and \[ R_z = U \Sigma^2 U^H \]

When estimated from data \[ Z = U \Sigma V^H \]

\[ |v_j d^H|^2 \] for \( j = 1 \ldots N - 1 \)
The MWF is a successive decomposition of the GSLC

\[ h_1 = \frac{r_{d_0 z_1}}{\sqrt{r_{d_0 z_1}^H r_{d_0 z_1}}} \quad h_1 \perp G_1 \]

\( G_1 \) is then decomposed

\[ h_2 = \frac{r_{d_1 z_2}}{\sqrt{r_{d_1 z_2}^H r_{d_1 z_2}}} \]

A reduced rank filter is made by terminating the process at a stage less than N-1.
Adaptive Toy Example

\[ \Sigma^2 = \text{diag} [100 \hspace{5pt} 300 \hspace{5pt} 5000 \hspace{5pt} 1 \hspace{5pt} 1] \hspace{5pt} r = [50 \hspace{5pt} 30 \hspace{5pt} 50 \hspace{5pt} 0 \hspace{5pt} 0] \]

Assume \( \sigma_d^2 = 1 \) so, \( \rho = [0.5 \hspace{5pt} 0.1 \hspace{5pt} 0.01 \hspace{5pt} 0 \hspace{5pt} 0] \)

- **Known Covariance - All methods choose identically**
- **PCI must distinguish between 100 and 1**
- **CSM must distinguish between 0.01 and 0**
- **MWF must well estimated r**
Independent Test and Train

1. Simulate m vector snapshots of a Multivariate Gaussian with population correlation R with max possible SNR $S^T R^{-1} S$.

2. X_k

3. Transform to Beam Coordinates

4. Estimate $R_{Z_k}$

5. SVD

6. Prescribed rank p

7. Choose CSM Eigenvectors

8. Compute Weight Vector

9. Compute Theoretical SNR of W

10. Normalize SNR by $S^T R^{-1} S$.

11. Prescribed rank p

12. Choose PCI Eigenvectors

13. Compute Weight Vector

14. Compute Theoretical SNR of W

15. SNR(k)

16. Plot X

17. Plot Y

Repeat steps for each trial K.
Baseline Scenario

Scenario Used to Show Benefits of CSM for Known Covariance
Baseline Scenario PCI vs CSM

PCI significantly outperforms CSM on many realizations.

Looks similar to effects of subspace swaps.
Swapping?

PCI is a subspace method

CSM is an eigenvector method
Marginal Viewpoint

No swaps for PCI

Strong possibility of many swaps for CSM
When CSM Beat PCI

- There was no PCI Swap
- The cross covariance must also be estimated well for good performance
- Sometimes a CSM Swap will avoid a poor cross covariance estimate
Underestimated Rank

CSM does provide improved performance for underestimated rank
Baseline Scenario PCI vs MWF

Both methods have stable estimates and produce identical results.

Note that they are the same point by point.
Performance vs Rank

CSM and MWF performance for underestimated ranks is poor due to large power.

Methods that select based on correlation produce large weights for overestimated ranks.
Decreased JNR

Considerably more CSM swaps

Decreased JNR begins to effect MWF
Sample Size and Rank

CSM is Degraded for Significant Sample Sizes

CSM and MWF have improved performance for underestimated rank
Jammers Not in Nulls

CSM is Improved but Still Degraded

MWF has Slightly Less Performance at Small Sample Sizes
Rolloff Jammers

PCI has the Best Small Sample Performance

MWF has the Best Large Sample Performance
Test and Train on Same Data

• Given a set of snapshots, X. Does X contain signal or only noise.

• Performance Measure is $(S+N)/\text{avg}(N)$ at the output.

• Can assume the signal only occupies one snapshot without loss of generality.
Toy Example

Suppose there are two jammers with a level of 1000 and

\[ y = 0.01X_1 + 0.1X_2 + 0X_3 + 0X_4 \]

\[
\begin{bmatrix}
-90 \\
90 \\
110 \\
-110
\end{bmatrix}
\begin{bmatrix}
1000 & -1000 & 1000 & -1000 \\
-1000 & 1000 & 1000 & -1000 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0.0995 \\
0.9950 \\
0 \\
0
\end{bmatrix}

All Methods Will Choose the Same Subspace
Noise and Signal

\[ y = [10 \ 90 \ 110 \ -110] \quad x = \begin{bmatrix} 1000.3 & -999.2 & 1001 & -1002 \\ -998 & 1001 & 1002 & -1000.2 \\ -1 & -0.5 & 0.5 & 1 \\ 0.5 & 0.2 & 0.1 & -0.5 \end{bmatrix} \]

\[ r_{yx} = [35101 \ 75088 \ -27.5 \ 13.25] \]

\[ \rho_{yx} = [0.3898 \ 0.8341 \ -0.3865 \ 0.3970] \]

PCI Selection is Unaffected by the Presence of Signal

MWF shows little affect due to the Presence of Signal

CSM Makes an Error due to the Presence of Signal
Overestimated Rank and Signal

- PCI is unaffected by signal
- CSM would like to cancel the signal but can only use the vectors provided by the SVD
- MWF is free to construct vectors (within its framework) which cancel the signal
Single Jammer Scenario
PCI vs CSM

Signal Levels: None, 0, 12, 24 dB
PCI vs MWF

Signal Levels: None, 0, 12, 24 dB
Signal Level

CSM shows Reduced Performance with Signal Presence
Rank

MWF and CSM Suffer for Overestimated Ranks
Five Jammer Scenario

Same as Previous No Jammers in Nulls Scenario
Signal Level

CSM and MWF show Degraded Performance with Signal Level
Rank

CSM and MWF show Improvement over PCI for Underestimated Rank but Perform Poorly for Overestimation of Rank
ROC Curves

Receiver Operating Characteristic for 12dB Signal

$2^{15}$ Samples
Conclusions

- Insight by Use of Toy Examples Provides Rules of Thumb for Performance
- PCI Provides the Best Performance for Small Sample Sizes, Correct and Overestimated Ranks
- The MWF Provides Good Large Sample Performance and Small Sample Performance for Correct and Underestimated Ranks