Passive Differential Matched-field Depth Estimation of Moving Acoustic Sources

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Passive Moving Target Depth Estimation (MTDE)

**OBJECTIVE:** To discriminate submerged versus surface targets by exploiting changes in the spatial wavefront at the array due to multipath propagation from a moving source.

**BACKGROUND:**

- Conventional matched-field processors use a computational model to predict the relative phase and amplitude between multipath arrivals from a distant stationary source and thus are very sensitive to horizontal wavenumber differences multiplied by range.
- Target motion in classical MFP techniques is problematic since it tends to decorrelate multipath components over the observation times used with stationary source models.
- Previous work using moving sources has attempted to mitigate source motion effects by pre-processing so as to effectively remove target dynamics.
- Proposed work aims to exploit target dynamics to estimate source depth and range-rate without requiring the accurate environmental models required for range estimation.
- Joint depth-range-rate estimation should achieve robustness to environmental mismatch since it depends only on horizontal wavenumber differences multiplied by the *change* in target range.
Conventional MFP with a Vertical or Horizontal Array

A snapshot of tilted vertical linear array (TVLA) data can be modeled as:

\[ x_n = s_n U(\theta_s) a + \eta_n \]

where \([U(\theta_s)]_{ml} = \phi_I(z_m)e^{-jk_m d} \sin \gamma \sin \theta_s, [a(r_S, z_s)]_l = \phi_I(z_s)e^{-jk_s r_s},\]

\(\theta_s, r_s, z_s\) are source bearing, range, depth, and \(\gamma\) is array tilt.

Full 3-D range-depth-bearing adaptive MFP requires accurate prediction of \((k_l - k_j)r_s\) which is difficult for large range, sufficient observation time over which the source can be considered stationary, and a search over 3 variables.
Some Previous Depth Estimation Approaches

• Averaging a 3-D MV surface over range may be computationally intensive. Further, matrix inversion prior to averaging can be statistically unstable.

• Matching the normal mode power distribution versus hypothesized target depth requires near orthogonality between modes at the array.

• The MV adaptive beamformer with extended range constraints (MV-ERC) consists of widening the range mainlobe so that bearing-depth estimation can be performed in coarse range bands.

• Desensitizing the adaptive beamformer to target range variation permits the use of longer observation times for more stable CSDM estimation.

• The ambiguity surface for the MV-ERC beamformer is given by:

\[ Z_{ERC}(r,z) = e_1^\dagger (H(r,z)^\dagger R_x H(r,z))^{-1} e_1 \]

where \( e_1 = [1,0,...,0]^\dagger \), where \( H(r,z) \) are the dominant eigenvectors of

\[ \frac{1}{N} \sum_{k=1}^{N} d(r + \Delta_k, z) d(r + \Delta_k, z)^\dagger \] and \( \Delta_k, k = 1,...,N \) defines a coarse range band around \( r \).
MV-ERC Matched-field Beamforming Results

- Typical simulated ambiguity surface for 8-tonal SWELLEX-96 TVLA event S5 5/10/96 2332 Z. Obs. Time = 54 s, SR=5800 m, SD=49 m, and SB = asin(0.34).

- Typical real ambiguity surface for 8-tonal SWELLEX-96 TVLA event S5 5/10/96 2332 Z. Obs. Time = 54 s, SR=6100 m, SD=56 m, and SB=asin(0.33).
Fundamental Depth Estimation Considerations

- Robust bearing-depth discrimination without range could in principle be obtained by treating modal phase terms as nuisance parameters, i.e. \[ a(r_s, z_s) = \phi_l(z_s) e^{-j\theta_l} \]
- Cramer-Rao Lower Bound (CRLB) on source depth (left) for the known (“exact”) versus unknown (“modified”) modal phase suggests depth estimation without range possible.
- Sampled modal eigenfunctions (right) sampled at two depths illustrate ambiguity in estimating modal phases jointly with source depth without target motion.
A Dynamical Model for Passive Depth Estimation

- Idea is to exploit modal phase trajectory under a constant range-rate hypothesis in order to jointly estimate target range-rate and depth.

- Letting the complex range-dependent modal amplitudes of a source for snapshot $k$ be denoted $x_k$, the relative changes in modal phase from snapshot-to-snapshot impose a Markov state update:

$$ x_k = A_k(\dot{r})x_{k-1} + \nu_k $$

where $A_k(\dot{r}) \equiv diag(e^{jk_l(\eta_{k-1})}) = diag(e^{jk_l(\eta_{k-1})})$ and the additive process noise approximately accounts for horizontal wavenumber uncertainties.

- The spatial wavefront at the array, $y_k$, at narrowband snapshot $k$, is then obtained by taking the sum of the normal modes multiplied by an i.i.d. zero-mean Gaussian random scalar, $S_k$:

$$ y_k = s_kU(\theta_s)\Phi(z_s)x_k + \eta_k $$

where $\Phi(z_s) = diag(\phi_l(z_s))$ and $\eta_k$ represents additive noise.
A Recursive Resampled Bayesian Estimate for Depth

- The non-linear depth-range-rate estimation problem can be solved by representing the posterior density function of the state by a set of random samples, rather than a continuous function over some high dimensional state space.

- For example, suppose at step $k$, random samples, $x_{k-1}^*(i), i = 1,\ldots,N,$ are available from $p(x_{k-1}|y_1,\ldots,y_{k-1})$. Then samples, $x_k^*(i)$ from $p(x_k|y_1,\ldots,y_{k-1})$ can be obtained using these samples as input to the state equation together with samples, $E_{g_s}$, drawn from its known distribution.

- The updated posterior density can then be approximated at each sample, $x_k^*(i)$, by forming:

$$q_i = \frac{p(y_k|x_k^*(i))}{\sum_{j=1}^{N} p(y_k|x_k^*(i))}$$

- Samples, $x_k^*(i), i = 1,\ldots,N,$ can now be obtained by bootstrap resampling $N$ times from the discrete distribution defined such that for any $j$, $Pr\{x_k(j) = x_k^*(i)\} = q_i$. The conditional mean of the depth parameter, $z_k$, can then be estimated by averaging these bootstrap samples.

- These steps are repeated for each range step to obtain a recursive estimate.

- For passive sonar, $p(y_k|x_k, z_s)$ is zero-mean Gaussian with covariance $R_k = \sigma^2_s U \Phi(z_s) x_k x_k^* \Phi(z_s)^+ U^+ + \sigma^2 I$ as in conventional models.
Sequential Importance Sampling Illustration

- Illustration clockwise from upper left of random samples from prior, weighting by likelihood function, Monte Carlo re-sampling from updated posterior, prediction using random samples of state noise.
Recursive Bayesian Passive MTDE Summary

- State vector includes unit-magnitude modal coefficients and range-rate with uniform prior.
- State transition density assumes multiplicative modal phase noise.
- Conditional density of data snapshot given state vector is zero-mean complex Gaussian.
- Estimates of depth-range-rate likelihood achieved using sequential importance sampling.

\[ \hat{y}(k) \]
\[ \hat{p}(x_k \mid y_1, \ldots, y_{k-1}, z_s) \]
\[ \hat{p}(x_k \mid y_1, \ldots, y_k, z_s) \]
\[ \hat{p}(y(1), \ldots, y(k) \mid z_s, \hat{r}) \]
Mismatched Range-Independent Simulation

• To illustrate passive moving target depth estimation, a simple simulation was performed using a normal mode model of a 15-mode Pekeris waveguide with a 100 m. waveguide.

• Simulation of 0 dB SNR targets, at 2 m. and 20 m. depths with 2 m/s range-rate, received at a water-column spanning 23 sensor vertical array using ~50 narrowband snapshots was considered.

• Large environmental uncertainty was simulated by adding independent uniform random variables to the Pekeris vertical wavenumbers, $k_z = k_z^0 + \Delta k_z$, where $\Delta k_z = U(-0.45\pi/h,0.45\pi/h)$ used to compute both horizontal wavenumbers and modal depth eigenfunctions.
Mismatched Conventional MFP for a Submerged Source

- Example conventional (aka “Bartlett”) ambiguity surface (left) for moving source at 20 m. depth illustrates extreme ambiguity problem.

- Log-histogram (right) of MFP estimates using 100 Monte Carlo trials illustrates poor range estimation and mediocre depth estimation performance over \( \Delta k_z = U(-0.45\pi/h,0.45\pi/h) \)
Mismatched Passive MTDE for a Submerged Source

- Example depth-range-rate log-likelihood surface (left) for moving source at 20 m. depth illustrates depth ambiguity with excellent range-rate estimation.

- Log-histogram of MTDE (right) over 100 Monte Carlo trials illustrating joint depth-range-rate estimation performance over $\Delta k_z = U(-0.45\pi/h, 0.45\pi/h)$
Mismatched Conventional MFP for Near-Surface Source

- Example conventional (aka “Bartlett”) ambiguity surface (left) for moving source at 2 m. depth and 2 m/s range-rate illustrates extreme range ambiguity problem.

- Log-histogram (right) of MFP estimates using 100 Monte Carlo trials illustrates poor range estimation but good depth estimation performance over \( \Delta k_z = U(-0.45\pi/h,0.45\pi/h) \)

- In practice, detection of a moving source from uncorrelated surface-based noise seriously limits the use of conventional MFP for depth classification.
Mismatched Passive MTDE for a Near-Surface Source

- Example depth-range-rate log-likelihood surface (left) for moving source at 2 m. depth illustrates depth estimation comparable to MFP with excellent range-rate estimation.

- Log-histogram of MTDE (right) over 100 Monte Carlo trials illustrates good range-rate estimation and depth estimation performance over $\Delta k_z = U(0, 0.9\pi/h)$

- Ability of passive MTDE to discriminate constant range-rate sources may facilitate detection of targets at depth from surface shipping with different range-rates.
Comparison of Depth Estimation Performance

- Histograms of MTDE (upper left) vs. conventional (lower left) for submerged source with $\Delta k_z = U(-0.45\pi/h, 0.45\pi/h)$ mismatch indicates moderately improved performance.

- Probability of correct depth localization (notwithstanding range, range-rate, or predicted depth ambiguity) compares performance for surface versus submerged source.

- Joint PCL for MFP range-depth estimate versus MTDE range-rate-depth estimate expected to show significant improvement for latter approach in mismatched channels.
Conclusions and Future Work

• Moving target depth estimation (MTDE) shows potential as a classification tool for passive discrimination of sources in the water column versus surface ships.

• By exploiting target motion, MTDE jointly estimates depth and range-rate avoiding severe range ambiguity problems of conventional MFP in mismatched conditions.

• Current sequential importance sampling approach for solving MTDE requires further development for operation at lower signal-to-noise ratios.

• MTDE framework may provide an alternative detection strategy by considering bearing-range-rate-depth likelihood surfaces.

• Current work includes evaluating MTDE with real SACLANT and SWELLEX data.

• Straightforward broadband passive implementation can be achieved by incoherently summing a posteriori probabilities across the frequency band.