Adaptive Beamformer Orthogonal Rejection Test (ABORT)

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Detection Ambiguities in Adaptive Arrays

Detect Mainlobe Targets

Blank Detects due to Sidelobe Targets or Clutter Discretes

Mitigate Interference / Jamming
Outline

• Previous Work on Adaptive Detection
• “ABORT” - a new Adaptive Detection Algorithm
• Analysis and Performance Results
• Summary
Adaptive Detection Problem

HYPOTHESIS TESTING

Complex Test Vector \( x = \begin{cases} n & \text{noise only hypothesis } H_0 \\ a v + n & \text{signal in noise hypothesis } H_1 \end{cases} \)

ASSUMPTIONS

<table>
<thead>
<tr>
<th>Unknown</th>
<th>Complex Signal Scalar</th>
<th>1×1</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise Covariance Matrix</td>
<td>N×N</td>
<td>( R = E{nn^H} )</td>
<td>Zero-mean complex Gaussian noise</td>
</tr>
<tr>
<td>Given</td>
<td>Test Vector</td>
<td>N×1</td>
<td>x</td>
</tr>
<tr>
<td>IID Training Vectors</td>
<td>N×1</td>
<td>( x_i ; i = 1, \ldots, K )</td>
<td>Noise-only samples ( (H_0) )</td>
</tr>
<tr>
<td>Signal Vector</td>
<td>N×1</td>
<td>v</td>
<td></td>
</tr>
</tbody>
</table>
Earlier Adaptive Detection Tests

Adaptive Matched Filter (AMF)
Robey, et. al.

\[ t_{AMF} = \frac{v_S^H S^{-1} x}{v_S^H S^{-1} v_S} \]

H_1 > \eta_{AMF}
H_0

Generalized Likelihood Ratio Test (GLRT)
Kelly

\[ t_{GLRT} = \frac{t_{AMF}}{1 + x^H S^{-1} x} \]

H_1 > \eta_{GLRT}
H_0

Adaptive Coherency Estimator (ACE)
Conte, Scharf

\[ t_{ACE} = \frac{t_{AMF}}{x^H S^{-1} x} \]

H_1 > \eta_{ACE}
H_0

DEFINITIONS
Steering Vector \( v_S \)
Test Vector \( x \)
Sample Covariance Matrix \( S = \sum_{k=1}^{K} x_k x_k^H \)
Signal Mismatch and SINR

Signal Vector

$$x = a v + n$$

Steering Vector

$$t_{AMF} = \frac{|v_s^H S^{-1} x|^2}{v_s^H S^{-1} v_s}$$

Signal mismatch angle $\theta$, measured in whitened N-space

$$\cos^2 \theta = \frac{|v_s^H R^{-1} v|^2}{(v_s^H R^{-1} v_s)(v_s^H R^{-1} v)}$$

- Matched Case
  $$v = v_s$$
  $$\cos^2 \theta = 1$$

- Mainbeam Detector Performance
  $$v \approx v_s$$
  $$\alpha \leq \cos^2 \theta < 1$$

- Mismatched Case
  $$v \neq v_s$$
  $$0 \leq \cos^2 \theta < 1$$

- Sidelobe Detector Performance

Signal to Interference-plus-Noise Ratio

$$\text{SINR} = |a|^2 v^H R^{-1} v$$
Desirable Detection Performance

Probability of Detection vs SINR

Ideal Performance
Realistic Performance
Mesa Plot Comparison

\[ N = 5, \ K = 25, \ P_{FA} = 10^{-4} \]

- **AMF**
  - Poor mismatch signal discrimination
  - Good matched signal SNR performance

- **ACE**
  - Good mismatch signal discrimination
  - Poor matched signal SNR performance

- **GLRT**
  - Mismatch signal performance between AMF and ACE
  - Matched signal SNR performance comparable to AMF
Adaptive Sidelobe Blanker

$N = 5, \ K = 25$

Test Vectors

AMF

Yes

$\mathbf{t_{AMF}} > \eta_{AMF}$

No

$H_0$

ACE

Yes

$\mathbf{t_{ACE}} > \eta_{ACE}$

No

$H_0$

$H_1$

† ASAP 1996 - 1998
Kreithen, Baranoski, and Richmond

$P_{FA}$

$10^{-6}$

$10^{-5}$

$10^{-4}$

$10^{-3}$

$\cos^2(\theta)$

$0.0001$

$0.001$

$0.01$

$0.1$

$0.5$

$0.9$

$\mathbf{\text{ASB Threshold Pairs}}$

$\mathbf{\text{SINR [dB]}}$
Adaptive Sidelobe Blanker

\[ N = 5, \ K = 25 \]

Test Vectors

\[ t_{\text{AMF}} > \eta_{\text{AMF}} \]

Yes

\[ t_{\text{ACE}} > \eta_{\text{ACE}} \]

Yes

\[ \mathcal{H}_1 \]

No

\[ \mathcal{H}_0 \]

\[ \mathcal{H}_0 \]

† ASAP 1996 - 1998
Kreithen, Baranoski, and Richmond

ASB Threshold Pairs

\[ P_{\text{FA}} \]
\[ 10^{-6} \]
\[ 10^{-5} \]
\[ 10^{-4} \]
\[ 10^{-3} \]

\[ \cos^2(\theta) \]

\[ \text{SINR [dB]} \]

0 1 0 2 0 3 0
0
0.2
0.4
0.6
0.8
1
0 10 20 30
K \eta_{\text{AMF}}
Adaptive Sidelobe Blanker

N = 5, K = 25

Test Vectors

AMF

Yes

ACE

Yes

H1

No

No

H0

H0

‡ ASAP 1996 - 1998
Kreithen, Baranoski, and Richmond

ASB Threshold Pairs

PFA

10^{-6}

10^{-5}

10^{-4}

10^{-3}

ηACE

0.6

0.4

0.2

0.8

0.6

0.4

0.2

0.8

0.6

0.4

0.2

K ηAMF

0

10

20

30

SINR [dB]

cos^2(θ)

0

0.2

0.4

0.6

0.8

1

0.001

0.0001

0.1

0.5

0.9

‡ ASAP2000-11
NBP 03/29/2000
Adaptive Sidelobe Blanker

\[ N = 5, \quad K = 25 \]

Test Vectors

\[ t_{\text{AMF}} > \eta_{\text{AMF}} \]

\[ \text{No} \]

\[ t_{\text{ACE}} > \eta_{\text{ACE}} \]

\[ \text{Yes} \]

\[ \text{Yes} \]

\[ H_1 \]

\[ H_0 \]

† ASAP 1996 - 1998
Kreithen, Baranoski, and Richmond
Adaptive Sidelobe Blanker

\[ N = 5, \ K = 25 \]

Test Vectors

- \( t_{\text{AMF}} > \eta_{\text{AMF}} \) Yes
  - AMF
  - \( \text{No} \)
  - \( H_0 \)

- \( t_{\text{ACE}} > \eta_{\text{ACE}} \) Yes
  - ACE
  - \( \text{No} \)
  - \( H_0 \)

\[ \eta \text{ ACE} > \eta \text{ AMF} > \eta \]

\[ \text{SINR [dB]} \]

\[ \cos^2(\theta) \]

\[ P_{\text{FA}} \]

- \( 10^{-6} \)
- \( 10^{-5} \)
- \( 10^{-4} \)
- \( 10^{-3} \)

\[ \text{ASB Threshold Pairs} \]

\[ \text{† ASAP 1996 - 1998} \]

Kreithen, Baranoski, and Richmond
Adaptive Sidelobe Blanker †

\[
N = 5, \quad K = 25
\]

Test Vectors

- \( t_{AMF} > \eta_{AMF} \)
  - Yes \( \rightarrow H_1 \)
  - No \( \rightarrow H_0 \)

- \( t_{ACE} > \eta_{ACE} \)
  - Yes \( \rightarrow H_1 \)
  - No \( \rightarrow H_0 \)

† ASAP 1996 - 1998
Kreithen, Baranoski, and Richmond

\[ \cos^2(\theta) \]

\[ P_{FA} \]

- \( 10^{-6} \)
- \( 10^{-5} \)
- \( 10^{-4} \)
- \( 10^{-3} \)

\[ \eta_{ACE} \]

\[ K \eta_{AMF} \]
Outline

• Previous Work on Adaptive Detection

• “ABORT” - a new Adaptive Detection Algorithm

• Analysis and Performance Results

• Summary
Assumptions for Two-step Detection

Detection Hypotheses (Test 1)

\[
x = \begin{cases} 
  n & \text{noise only hypothesis } H_0 \\
  a\mathbf{v} + n & \text{signal in noise hypothesis } H_1 
\end{cases}
\]

Sidelobe Blanking Hypotheses (Test 2)

\[
x = \begin{cases} 
  a\mathbf{v}_\perp + n & \text{orthogonal signal in noise } H_0 \\
  a\mathbf{v} + n & \text{signal in noise } H_1 
\end{cases}
\]

Assumptions

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<tr>
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<th>IID Training Vectors</th>
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<tr>
<td>Complex Signal Scalar</td>
<td>1 \times 1</td>
<td>\mathbf{a}</td>
<td>\mathbf{N} \times \mathbf{N}</td>
<td>R = \mathbb{E} { \mathbf{n}\mathbf{n}^H }</td>
<td>\mathbf{N} \times 1</td>
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<td>Noise Covariance Matrix</td>
<td>\mathbf{N} \times \mathbf{N}</td>
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<td>Noise-only samples</td>
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Zero-mean complex Gaussian noise
Likelihood Ratio Formulation for Test 2

\[
x = \begin{cases} 
  \mathbf{a} \mathbf{v}^\perp + \mathbf{n} & \text{orthogonal signal in noise } H_0 \\
  \mathbf{a} \mathbf{v} + \mathbf{n} & \text{signal in noise } H_1 
\end{cases}
\]

\[
x_k = \mathbf{n} ; \ k = 1, \ldots, K \quad \text{noise only}
\]

IID Training Vectors

\[
f(\mathbf{R}, \mathbf{a} \mathbf{y} | \mathbf{x}, \mathbf{x}_1, \ldots, \mathbf{x}_K) = \left[ \frac{\exp\left\{- \text{tr} \left( \mathbf{R}^{-1} \mathbf{T} \right) \right\}}{\pi^N | \mathbf{R} |} \right]^{K+1}
\]

Likelihood Function

\[
\mathbf{T} \equiv \frac{1}{K+1} \left\{ (\mathbf{x} - \mathbf{a} \mathbf{y})^H (\mathbf{x} - \mathbf{a} \mathbf{y}) + \sum_{k=1}^{K} \mathbf{x}_k \mathbf{x}_k^H \right\}
\]

\[
y \equiv \begin{cases} 
  \mathbf{v}^\perp & \text{under } H_0 \\
  \mathbf{v} & \text{under } H_1 
\end{cases}
\]

GLR Approach: Maximize likelihood function over unknown parameters under each hypothesis
Likelihood Ratio Formulation for Test 2 continued

\[ f(R, ay \mid x, x_1, \ldots, x_K) = \left[ \frac{\exp\{- \text{tr} (R^{-1} T)\}}{\pi^N \mid R \mid} \right]^{K+1} \]

Likelihood Function

\[ m = \max_R \{ f(R, a) \} = \left\{ (e\pi)^N \mid T \mid \right\}^{-K-1} \]

Maximize over unknown covariance matrix

\[ m_0 = \max_a \{ m \} = \left\{ \mid S \mid (1 + t_{AMF}) \right\}^{-K-1} \]

Maximize over unknown orthogonal signal (\(H_0\))

\[ m_1 = \max_a \{ m \} = \left\{ \mid S \mid (1 + x^H S^{-1} x - t_{AMF}) \right\}^{-K-1} \]

Maximize over unknown signal multiplier (\(H_1\))

Form Likelihood Ratio Test

\[ t = \left( \frac{m_1}{m_0} \right)^{\frac{1}{K+1}} = \frac{1 + t_{AMF}}{(1 + x^H S^{-1} x - t_{AMF})} \]

\(H_1 \begin{cases} \text{if } > \eta \\ \text{if } < \eta \end{cases}\)

\(H_0\)

Equivalent Form of Test

\[ \tilde{t} = \frac{1 + t_{AMF}}{(2 + x^H S^{-1} x)} \]

\(H_1 \begin{cases} \text{if } > \tilde{\eta} \\ \text{if } < \tilde{\eta} \end{cases}\)

\(H_0\)
Efficient Procedure for ABORT

Test Vectors → AMF $t_{AMF} > \eta_{AMF}$ → Yes

Test Vectors → ABORT $\bar{t} > \bar{\eta}$ → Yes

Yes

No

No

$H_0$

$H_0$

$H_1$

Test 1

$t_{AMF} = \frac{v^H S^{-1} x}{v^H S^{-1} v} > \eta_{AMF}$

Test each observation $x$

Test 2

$\bar{t} = \frac{1 + t_{AMF}}{2 + x^H S^{-1} x} > \bar{\eta}$

Test observations that pass previous test

Note: If we select thresholds such that $\eta_{AMF} \leq 1 / (\bar{\eta} - 1)$

Then the 2-step procedure has detection performance equivalent to Test 2
Outline

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Example Problem

- Developed analytical expressions for $P_D$ and $P_{FA}$
  - Details provided in proceedings
- Verified $P_D$ with 10,000 independent Monte Carlo trials
- $N = 5$ channels
- $K = 25$ training vectors
- Choose thresholds so that $P_{FA} = 10^{-4}$
- Vary SINR over $0 \text{ dB} \leq \text{SINR} \leq 25 \text{ dB}$
- Vary mismatch angle over $0 \leq \cos^2 \theta \leq 1$
Mesa Plot for ABORT

N = 5, K = 25, \( P_{FA} = 10^{-4} \)

Mismatch signal performance is between GLRT and ACE
Matched signal performance comparable to GLRT and AMF
Mesa Plot Comparison

\( N = 5, \ K = 25, \ P_{FA} = 10^{-4} \)

- AMF
- GLRT
- ACE
- ABORT
- ASB

Cos\(^2(\theta)\) vs. SINR [dB]

ASB Threshold Pairs

\( \eta_{ACE} \) vs. \( K \eta_{AMF} \)
Mesa Plot Comparison

$N = 5, \ K = 25, \ P_{FA} = 10^{-4}$
Mesa Plot Comparison

$N = 5, \ K = 25, \ P_{FA} = 10^{-4}$

- AMF
- GLRT
- ACE
- ABORT
- ASB

$\cos^2(\theta)$ vs. SINR [dB]

ASB Threshold Pairs

$\eta_{\text{ACE}}$ vs. $K \eta_{\text{AMF}}$
Mesa Plot Comparison

$N = 5, \ K = 25, \ P_{FA} = 10^{-4}$
Mesa Plot Comparison

$N = 5, \; K = 25, \; P_{FA} = 10^{-4}$
Mesa Plot Comparison

\( N = 5, \ K = 25, \ P_{FA} = 10^{-4} \)

AMF

GLRT

ACE

ABORT

ASB

\( \cos^2(\theta) \)

\( \text{SINR [dB]} \)

\( \cos^2(\theta) \)

\( \text{SINR [dB]} \)

\( \cos^2(\theta) \)

\( \text{SINR [dB]} \)

\( \eta_{\text{ACE}} \)

\( K \eta_{\text{AMF}} \)
Slice through Mesa Plot at SINR = 20dB

$N = 5$, $K = 25$, $P_{FA} = 10^{-4}$

ABORT mismatch performance between GLRT and ACE
Slice through mesa plot at $\cos^2 \theta = 1$

$N = 5, \ K = 25, \ P_{FA} = 10^{-4}$

AMF, GLRT and ABORT have similar matched signal performance

Approximately 5 dB in SINR Loss with ACE
Detector Performance Overview
Summary

• Developed and analyzed the Adaptive Beamformer Orthogonal Rejection Test (ABORT)
  – Generalized Likelihood Ratio Test to discriminate signals in one subspace from signals in an orthogonal subspace

• ABORT compares favorably relative to one-step tests (AMF, ACE and GLRT) and a two-step test (ASB)
  – Improved sidelobe rejection performance relative to AMF, GLRT
  – Mainlobe detection performance commensurate with AMF and GLRT
  – ABORT provides a good compromise of mainbeam detection and sidelobe rejection without sensitivity to target SINR (as with ASB)

• Implement as second test in a two-test approach (AMF followed by ABORT) for computational efficiency